

Econ 522: Intermediate Macroeconomics, Fall 2017
Chapter 3 Classical Model Practice Problems

1. **Explain what determines the amount of output an economy produces?**

The factors of production and the available production technology determine the amount of output an economy can produce. Factors of production are the inputs used to produce goods and services (capital, and labor). Production technology determines how much output can be produced from any given amounts of these inputs (described by the production function). Increasing one of the factors of production or improving technology will increase the economy's output.

2. **Explain the rule that competitive, profit-maximizing firms follow when they decide how much capital and labor to hire.**

See textbook for explanation. A competitive profit-maximizing firm hires labor until the marginal product of labor equals the real wage, and similarly hires capital until the marginal product of capital equals the real rental rate.

3. **Returns to Scale.** Do the following production functions exhibit increasing, constant, or decreasing returns to scale?

(a) $F(\mathbf{K}, \mathbf{L}) = \mathbf{K}^{1/2} \mathbf{L}^{1/2}$

Constant returns to scale.

$$\begin{aligned} F(zK, zL) &= (zK)^{1/2} (zL)^{1/2} \\ &= z^{1/2} K^{1/2} z^{1/2} L^{1/2} \\ &= z^{1/2} z^{1/2} K^{1/2} L^{1/2} \\ &= z^{1/2 + 1/2} K^{1/2} L^{1/2} \\ &= z F(K, L) \end{aligned}$$

(b) $F(\mathbf{K}, \mathbf{L}) = \mathbf{K}^{1/2} + \mathbf{L}^{1/2}$

Decreasing returns to scale.

$$\begin{aligned} F(zK, zL) &= (zK)^{1/2} + (zL)^{1/2} \\ &= z^{1/2} (K^{1/2} + L^{1/2}) \\ &= z^{1/2} F(K, L) \\ &< z F(K, L) \text{ if } z > 1 \end{aligned}$$

(c) $F(\mathbf{K}, \mathbf{L}) = \frac{\mathbf{K}^2}{\mathbf{L}}$

4. **Diminishing Returns.** Which of the following production functions have diminishing marginal returns to labor?

(a) $F(K, L) = 2K + 15L$

The $MPL = 15$ for all values of L , so this production function does not have diminishing returns to labor.

(b) $F(K, L) = \sqrt{KL}$

As L rises the MPL falls, so this production function does have diminishing returns to labor.

(c) $F(K, L) = 2\sqrt{K} + 15\sqrt{L}$

As L rises the MPL falls, so this production function does have diminishing returns to labor.

5. If a natural disaster killed a substantial portion of a nation's population, and there were no changes in the available capital or level of technology, what would the classical model of production predict will happen to the real wage of workers in that nation? Explain the underlying logic leading to that prediction.

6. Using the chapter 3 model of production, predict the impact on the real wage and the real rental price of capital of each of the following events:

- (a) **A wave of immigration increases the labor force.**

$\uparrow$ in the labor force $\Rightarrow \downarrow MPL$ (because of diminishing returns to L) $\Rightarrow \downarrow$ real wage (neoclassical theory of distribution: real wage equals marginal product of labor)

- The $\uparrow$ in L holding all else constant, moves the economy to a different point on the production function. At the point, because of diminishing returns to L the MPL is lower.

With a Cobb Douglas production function, $\uparrow L \Rightarrow \uparrow MPK \Rightarrow \uparrow$ real rental price of capital. With more workers, K will be used more intensively and will be more productive.

- Recall from the equation for MPK that it depended on L . One way to see the impact is to use example numbers with that equation. Pick two different values for L and see what the difference in MPK is.
- More intuitively, the idea is along the lines of – if everyone in a small office uses the same stapler, that stapler is more productive in the sense that it will be used to staple more than it would if there were also a second stapler shared among the office.

- (b) **An earthquake destroys some of the capital stock.**

$\downarrow$ in capital stock (with no change in labor force) $\Rightarrow \uparrow MPK \Rightarrow \uparrow$ real rental rate (because real rental rate equals MPK)

With a Cobb Douglas production function, $\downarrow K \Rightarrow \downarrow MPL \Rightarrow \downarrow$ real wage. With less K , each worker becomes less productive.

- (c) **A technological advance improves the production function.**

Technological advance improving the production would probably $\uparrow$ the MP of both K and L . Then both real wage and the real rental rate would increase.

- (d) **High inflation doubles the prices of all factors and outputs in the economy.**

No impact on the real wage or on the real rental price of capital if both nominal wages and the nominal rental price of K are also doubled.

7. Suppose the production for a nation is $Y = K^{0.5}L^{0.5}$ where K is the amount of land and L is the amount of labor. If the economy begins with 100 units of land and 100 units of labor, find numerical answers to each of the following questions:

- (a) **How much output does the economy produce?**

Substitute the given values for L and K into the production function to get the amount of output.

$$Y = K^{0.5}L^{0.5} = 100^{0.5}100^{0.5} = 100$$

- (b) **What are the wage rate and rental price of land?**

The real wage and the real rental rate will equal the marginal products.

$$MPL = (1 - \alpha)K^{\alpha}L^{-\alpha}$$

$$MPK = \alpha K^{\alpha-1}L^{1-\alpha}$$

Here $\alpha = 0.5$. Substitute in L and K to find the marginal products. $MPL = 0.5$ and $MPK = 0.5$.

- (c) **What share of output does labor receive?**

The quantity of labor times the real wage rate.

$$L \times \frac{W}{P} = 100 \times 0.5 = 50$$

- (d) **If a plague kills exactly half of the population, what is the new level of output?**

New output level is 70.71. So Y decreases.

- (e) **What is the new wage and rental price of land?**

New wage is 0.71. New rental price of land is 0.35.

- (f) **What share of output does labor receive now?**

Labor now receives 35.36.

8. **Explain what determines consumption and investment?**

In the chapter 3 model, consumption is determined by disposable income (income minus taxes), the marginal propensity to consume, and potentially an autonomous (subsistence) level of consumption.

Investment is determined by the real interest rate.

9. **Explain the difference between government purchases and transfer payments. Give two examples of each.**

Government purchases are when the government spends money on goods and services. Examples include spending to build roads, weapons acquisition, and building schools. Transfer payments are payments that are not in exchange for goods and services. Examples include subsidies to farmers, food stamps, and Social Security payments to the elderly.

10. **What adjusts to make demand for the economy's output of goods and services equal the supply?**

The real interest rate.

11. **Explain what happens to consumption, investment, and the interest rate when the government increases taxes.**

Summary: $\uparrow T \Rightarrow \downarrow (Y - T) \Rightarrow \downarrow C$ & private savings. And, $\uparrow T \Rightarrow \uparrow$ public savings ($G - T$). As long as $MPC < 1$, $\downarrow$ in private savings $< \uparrow$ public savings, so $\Rightarrow \uparrow$ total savings $\Rightarrow \downarrow r \Rightarrow \uparrow I$.

Explanation: When the government increases taxes, disposable income falls, and therefore consumption falls as well. The decrease in consumption equals the amount taxes increase times the marginal propensity to consume MPC . The higher the MPC is, the greater is the negative effect of the tax increase on consumption. Private savings, the difference between disposable income and consumption, will also decrease. The decrease in consumption is essentially offset by an increase in investment, which results from increased overall national savings driving down the real interest rate.

Public savings increases by the full amount of the tax increase. Disposable income decreases by the full amount of the tax increase, but only a portion of the decrease in disposable income will result in a decrease in private savings (the rest will decrease consumption). As a result, public savings will increase by more than private savings decreases, and total savings will increase.

12. **If the government raises taxes by \$100 billion and the marginal propensity to consume is 0.6, what happens to the following? (Do they increase or decrease, and by what amounts?)**
(a) **Public saving**

Public savings increases by \$100 billion. (Public savings = $T - G$, with no change in G , public savings increases by the full amount of the increase in T).

- (b) **Private saving**

Disposable income ($Y - T$) will decrease by \$100 billion. With $MPC = 0.6$, consumption will decrease by $0.6 \times \$100\text{billion} = \60 billion. The rest of the decline in disposable income will be the decrease in private savings $\rightarrow$ a decrease of \$40 billion.

(c) **National (total) saving**

Public savings increases by \$100 billion, and private savings decreases by \$40 billion. Combining those two changes, gives the change in total savings. Total savings increases by \$60 billion.

Note, the textbook uses the term ‘national savings’ for what refer to as ‘total savings.’ Both mean the same thing. I tend to prefer total, because national is easier to confuse with public.

(d) **Investment**

In equilibrium, investment will equal total savings, so investment will increase by the same amount as total savings does. (\$60 billion).

13. **Consider an economy described by the following equations:**

$$Y = C + I + G$$

$$Y = 5,000$$

$$G = 1,000$$

$$T = 1,000$$

$$C = 250 + 0.75(Y - T)$$

$$I = 1,000 - 50r$$

(a) **Compute private saving, public saving, and national saving.**

To find private savings, first figure out C :

$$C = 250 + 0.75(Y - T) = 250 + 0.75(5,000 - 1,000) = 3,250$$

Using C , and the given values for Y , T , and G :

$$\text{Priv. Sav.} = Y - T - C = 4,000 - 3,250 = \boxed{750}$$

$$\text{Pub. Sav.} = T - G = 1,000 - 1,000 = \boxed{0}$$

$$\text{Ttl Sav.} = \text{Priv. Sav.} + \text{Pub. Sav.} = 750 + 0 = \boxed{750}$$

(b) **Find the equilibrium interest rate.**

Equilibrium interest rate can be determined by setting the given equation for investment equal to savings, based on the idea that investment represents demand for loanable funds and savings represents supply of loanable funds. Then solve for the r that makes supply and demand equal to find the equilibrium rate.

$$I = 1,000 - 50r = 750$$

$$1,000 - 750 = 50r$$

$$r = \frac{1,000 - 750}{50} = \frac{250}{50} = \boxed{5}$$

- (c) If the government increases G to 1,250, what are the new levels of private saving, public saving, and national saving?

$$\text{Priv. Sav.} = Y - T - C = 4,000 - 3,250 = \boxed{750 \rightarrow \text{no change}}$$

$$\text{Pub. Sav.} = T - G = 1,000 - 1,250 = \boxed{-250 \rightarrow \downarrow \text{ by } \Delta G}$$

$$\text{Ttl Sav.} = \text{Priv. Sav.} + \text{Pub. Sav.} = 750 + (-250) = \boxed{500 \rightarrow \downarrow \text{ by } \Delta G}$$

- (d) Find the equilibrium interest rate.

$$1,000 - 50r = 500$$

$$1,000 - 500 = 50r$$

$$r = \frac{1,000 - 500}{50} = \frac{500}{50} = \boxed{10}$$

So the $\uparrow$ in G , decreased total S , which increased r . *Is that what you would expect from showing the change on a graph with supply and demand of loanable funds?*

14. Suppose the government increases taxes and government purchases by equal amounts. What happens to the interest rate and investment in response to this balanced-budget change? Explain how your answer depends on the marginal propensity to consume.