Econ 522: Intermediate Macroeconomics, Fall 2017

Chapters 8 and 9 Economic Growth Practice Problems

1. **Draw the Solow model diagram, label all of the curves and axes.**
   (a) **Label the steady state level of capital per worker k^*.**
   (b) **Pick a value greater than k^* and label it k_1. Explain in words what will happen to k over time if the economy starts at k_1?**
   (c) **Pick a value less than k^* and label it k_2. Explain in words what will happen to k over time if the economy starts at k_2?**

2. **Economy's aggregate production function: $Y = F(K,L) = \sqrt{K \times L} = K^{1/2}L^{1/2}$. Let $s = 0.3$, $\delta = 0.1$, and the initial value of $k = 4$. Assume there is no population growth.**
   (a) **Use the Solow model equation of motion ($\triangle k = s\ \mathrm{f}(k) - (\delta + n)k$) to solve for the steady-state values of k, y, and c.**

$$\underbrace{\triangle k}_{=0 \text{ in SS}} = s\ \mathrm{f}(k) - (\delta + n)\ k$$

So setting that equal to 0 describes the steady state. Plug in for f(k), s, n and δ. ($n = 0$ when you assume no population growth). Then solve for the steady state capital per work, k^*. The per-worker form of F(K,L) is f$(k) = \sqrt{k}$ (get this by dividing both the right and left hand side of the production function by L).

$$0 = 0.3\ \sqrt{k^*} - (0.1 + 0)\ k^*$$

$$0.1\ k^* = 0.3\ \sqrt{k^*}$$

$$\frac{k^*}{\sqrt{k^*}} = \frac{0.3}{0.1}$$

Squaring both sides of that equation gives: $\boxed{k^* = 9}$.

To find y when $k = 9$, plug 9 into f(k): $y = \sqrt{9} = \boxed{3}$. To find c, use the values of y and the savings rate. $s \times y = 0.3 \times 3 = 0.9$ goes toward savings. The rest, $y - s \times y = 3 - 0.3 \times 3 = 3 - 0.9 = \boxed{2.1}$.

   (b) **What happens when the savings rate increases to 0.4 (show on a graph and explain in words)? Find the steady state levels of capital per worker, and of output per worker. Are they higher or lower than before?**

If s increases to 0.4, steady state k, y and c will all increase. To find the new levels go through the same steps as above using the new savings rate). You should get $\boxed{\text{k=16}}$, $\boxed{\text{y=4}}$ and $\boxed{\text{c=2.8}}$.

3. **Country A and B both have the production function: $Y = K^{1/3}L^{2/3}$.**
   (a) **Does the production function have constant returns to scale?**

(b) **What is the per-worker production function?**

To find the per-worker production function, divide the aggregate production function by the number of workers, L.

$$\frac{Y}{L} = \frac{K^{1/3}L^{2/3}}{L} = \frac{K^{1/3}}{L^{1/3}}$$

Defining $y = Y/L$ and $k = K/L$, the above expression can be rewritten as:

$$y = k^{1/3}$$

(c) **Assume neither country has any population growth or technological progress and 20 percent of capital depreciates each year. Country A saves 10 percent of its output each year. Country B saves 30 percent of its output each year. Find steady state capital per-worker for both. Then find steady state levels of income per-worker and consumption per-worker.**

Country A: We know: $y = k^{1/3}$, $\delta = 0.2$, and $s_A = 0.1$.

First, find the steady state capital per-worker. In steady state, capital per-worker will not change over time. Without population growth, the change in capital per-worker $\triangle k$ is the amount of investment per-worker $s \times f(k)$, minus the amount of depreciation $\delta \times k$.

$$0 = s \times f(k^*) - \delta \times k^*$$

$$0 = 0.1 \times (k^*)^{1/3} - 0.2 \times k^*$$

$$\Rightarrow \quad k^* = (0.1/0.2)^{3/2} = \boxed{.35}$$

To find the steady state output per-worker, plug the steady state capital per-worker into the per-worker production function (which by definition tells you how much output per-worker you produce using a given amount of capital per-worker; when you plug in the steady state capital per-worker, you get the steady state output per-worker).

$$y^* = (k^*)^{1/3} = (.35)^{1/3} = \boxed{0.7}$$

All of the output per-worker will go to either savings or consumption. Using the savings rate, you can determine how much will be savings per-worker. The difference between the total amount of output per-worker and the amount saved per-worker will be consumption per-worker.

$$\text{amount of savings per-worker} = s \times y^* = 0.1 \times 0.7 = 0.07$$

$$c^* = y^* - \text{amount of savings per-worker} = 0.7 - 0.07 = \boxed{.63}$$

Country B: The only difference between A and B is the savings rate. For country B use $s_B = 0.3$ and go through the same steps as above. You should get: $k_B^* = 1.84$, $y_B^* = 1.22$, and $c_B^* = 0.86$.

(d) **If both start with capital stock per-worker of 1 (say in year 1), what are the levels of income per-worker and consumption per-worker in year 1? What will they be in year 2?**

This question asks you to solve for y and c when $k = 1$.

Country A: If $k = 1$, then $y_A = k^{1/3} = \boxed{1}$. To find c, use s to determine the amount saved per-worker, and subtract that from the total income per-worker (same process used above, and in a number of problems now).

$$c_A = y_A - s_A \times y_A = 1 - 0.1 \times 1 = \boxed{0.9}$$

Country B: For country B, you should get that $y_B = 1$ and $c_B = 0.7$. Notice that per-worker output is that same. This is because they are using the same technology, and for this part, the same amount of inputs. The per-worker consumption is different, because they save different portions of their incomes, and so have different amounts leftover after saving.

(e) **Show how the capital stock per-worker will evolve over time for both. Find y and c for each year. How long until B has higher consumption per worker than A?**

This part is more involved, but good for getting a better sense of what's going on in the Solow model. Start with $k = 1$. You know that for country A, $k^*_A = 0.35$, and for country B, $k^*_B = 1.84$ (see above). So one is starting above its steady state, while the other is starting below.

I'm going to drop the letter subscripts for now, and use number subscripts, starting at 1, to show how k is changing over time (1 for year 1, 2 for year 2, etc.). Use the underlined headings to distinguish which part applies to which country.

Country A:

$k_1 = 1$. From above, you know that $y_1 = 1$ and $c_1 = 0.9$. Given those numbers, $i_1 = s_A \times y_1$ will be 0.1 (because savings equals investment in a closed economy). Depreciation will be $\delta \times k_1 = 0.2 \times 1 = .20$. The change in k will be $\triangle k = i_1 - \delta \times k_1 = 0.1 - 0.2 = -0.1$. So in year 1, k will decrease by 0.1 (in year 2, you start with the level of k you had in year one minus 0.1).

k_2 will be k_1 adjusted for the change. So $k_2 = 1 - 0.1 = 0.9$. Given that level of k, you can solve for $y_2 = k_2^{1/3} = (0.9)^{1/3} = 0.97$. Applying the savings rate to y_2, you can find that $i_2 = s \times y_2 = 0.1 \times 0.97 = 0.097$, and then (as always) subtracting i_2 from y_2 gives c_2.

This process continues to repeat until the economy reaches the steady state. Below is a table of the approximate values you should get along the way if you continue solving for later years. The values are rounded to two decimal places.

year	k	y	c	i	δk	$\triangle k$
1	1	1	0.9	0.1	0.2	-0.1
2	0.9	0.97	0.87	0.1	0.18	-0.08
3	0.82	0.93	0.84	0.09	0.16	-0.07
4	0.75	0.91	0.82	0.09	0.15	-0.06
5	0.69	0.88	0.79	0.09	0.14	-0.05
6	0.64	0.86	0.78	0.09	0.13	-0.04
7	0.60	0.84	0.76	0.08	0.12	-0.04

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Country B:

$k_1 = 1$. From above, you know $y_1 = 1$ and $c_1 = 0.7$. Given those numbers, $i_1 = s_B \times y_1 = 0.3 \times 1$ will be 0.3 (because savings equals investment in a closed economy). Depreciation will be $\delta \times k_1 = 0.2 \times 1 = .20$. The change in k will be $\triangle k = i_1 - \delta \times k_1 = 0.3 - 0.2 = 0.1$. So in year 1, k will increase by 0.1 (in year 2, you start with the level of k you had in year one plus 0.1).

k_2 will be k_1 adjusted for the change. So $k_2 = 1 + 0.1 = 1.1$. Given that level of k, you can solve for $y_2 = k_2^{1/3} = (1.1)^{1/3} = 1.03$. Applying the savings rate to y_2, you can find that $i_2 = s \times y_2 = 0.3 \times 1.03 =$, and then subtracting i_2 from y_2 gives c_2.

As with country A, this process continues to repeat until the economy reaches the steady state. The table below has the different values for the first 7 years rounded to two decimal places.

year	k	y	c	i	δk	$\triangle k$
1	1	1	0.70	0.30	0.2	0.1
2	1.10	1.03	0.72	0.31	0.22	0.09
3	1.19	1.06	0.74	0.32	0.24	0.08
4	1.27	1.08	0.76	0.32	0.25	0.07
5	1.34	1.10	0.77	0.33	0.27	0.06
6	1.40	1.12	0.78	0.34	0.28	0.06
7	1.46	1.13	0.79	0.34	0.29	0.05

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Comparison:

Note that neither table goes all the way to the steady state. To get to the steady state, the value in the last columns has to go to 0. Also notice that when using actual numbers, the transition to a steady state can take quite a bit of time. Comparing the tables - Country B does not have higher per-worker consumption than Country A until the 7th year.

4. **Draw the Solow model diagram where the economy begins with a steady state level of capital per worker k^* that is below the golden rule level k_g^*. Label both k^* and k_g^*.**
   (a) **What change could cause the economy to move to k_g^*?**
   (b) **Describe the transition that would take place if that occured.**

5. **Draw a Solow model diagram with population growth, showing an economy in steady state. Use the graph to determine what happens to steady state capital per-worker and steady state income per-worker in response to each of the following exogenous changes.**
   (a) **Pessimism about nation's the future following elections, changes consumer preferences so as to increase the savings rate.**
   (b) **Changing weather patterns that result in more frequent and serious storms, increase the depreciation rate.**
   (c) **Better birth-control methods reduce the rate of population growth.**
   (d) **A one-time, permanent improvement in technology increases the amount of output that can be produced from any given amount of capital and labor.**